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Smearing effect in plane-wave matrix model

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Abstract: Motivated by the usual D2-D0 system, we consider a configuration composed of flat membrane and fuzzy sphere membrane in plane-wave matrix model, and investigate the interaction between them. The configuration is shown to lead to a non-trivial interaction potential, which indicates that the fuzzy sphere membrane really behaves like a graviton, giant graviton. Interestingly, the interaction is of r^{-3} type rather than r^{-5} type. We interpret it as the interaction incorporating the smearing effect due to the fact that the considered supersymmetric flat membrane should span and spin in four dimensional subspace of plane-wave geometry.

Keywords: Penrose limit and pp-wave background, M(atrix) Theories

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Contents

1 Introduction

The plane-wave or BMN matrix model [1] has been given by a mass deformation of the matrix model in flat spacetime, the BFSS matrix model [2], and preserves full eleven dimensional supersymmetry. It has been believed to describe the M-theory in maximally supersymmetric plane-wave background in the framework of the discrete light cone quantization (DLCQ).

One peculiar property of the plane-wave matrix model which has attracted much attention is that the supersymmetric fuzzy sphere membrane with finite size appears from the vacuum structure in this model [1, 3]. Although it is a configuration of membrane, it has been interpreted as a graviton, or more precisely a giant graviton because it has a size. After finding the presence of the fuzzy sphere membrane, there have been lots of work studying its properties and related issues from various viewpoints [3]–[9]. In the study of dynamical aspect, it has been shown that the fuzzy sphere behaves indeed like a graviton, and evidences about its interpretation as a giant graviton have been accumulated [10]–[17]. The thermodynamical aspect has also been considered, and the vacuum structure involving fuzzy sphere membranes at finite temperature has been investigated [18]–[22]. Upon a proper circle compactification, the plane-wave matrix model leads to the matrix string theory, which is related in the infrared limit to the free string theory in ten-dimensional plane wave background [23]–[27]. This string theory contains fuzzy spheres in its spectrum, whose various aspects also have been studied in refs. [28]–[31].

As for the dynamics of fuzzy sphere membrane, the research has been focused on the interaction between the fuzzy sphere membranes themselves. Interaction between different kinds of membranes or other supersymmetric objects in the plane-wave matrix model has not been considered seriously. In this paper, we are interested in the configuration composed of the fuzzy sphere and flat membranes, each of which is supersymmetric object, and investigate the interaction between them. If the interpretation of the fuzzy sphere membrane is definitely correct, the configuration may be thought to have a similarity with the usual D2-D0 system or more directly the membrane-graviton system [32] from eleven dimensional point of view. From this similarity, we may expect that the interaction computed from the path integration of the plane-wave matrix model around our configuration is the same up to numerical constant with that in the D2-D0 system. This expectation based on the well-known D2-D0 system motivates the present study. However, as we will show, our expectation is partially correct. The resulting interacting potential at large r distance gives one more evidence that the fuzzy sphere membrane behaves like a graviton, that is, a giant graviton, but has the r^{-3} type rather than the expected r^{-5} type. We will give an interpretation that the potential incorporates the delocalization or smearing effect due to the configuration of the supersymmetric flat membrane which should span and spin in four dimensional space.

The organization of this paper is as follows. In the next section, we will give an expansion of the plane-wave matrix model around a general classical background. The background configuration composed of the fuzzy sphere and flat membranes is presented in section 3. In section 4, the formal one-loop path integration of the plane-wave matrix model around the background configuration of section 3 is performed. From the result of path integration, the one-loop effective potential is obtained in section 5. Finally, we give the conclusion and discussion in section 6.

2 Plane-wave matrix model

The plane-wave or BMN matrix model [1] is a model for the microscopic description of the DLCQ M-theory in the eleven-dimensional *pp*-wave or plane-wave background [33], which is SO(3) × SO(6) symmetric and given by

$$\begin{aligned} ds^2 &= -2dx^+dx^- - \left(\sum_{i=1}^{3} \left(\frac{\mu}{3}\right)^2 (x^i)^2 + \sum_{a=4}^{9} \left(\frac{\mu}{6}\right)^2 (x^a)^2 \right) (dx^+)^2 + \sum_{I=1}^{9} \left(dx^I\right)^2 \,, \\ F_{+123} &= \mu \,, \end{aligned} \tag{2.1}$$

with the index notation $I = (i, a)$. This background is maximally supersymmetric and obtained by taking the Penrose limit to the eleven-dimensional AdS type geometries [34].

The plane-wave matrix model is basically composed of two parts. One part is the usual matrix model based on eleven-dimensional flat space-time, that is, the flat space matrix model, and another is a set of terms reflecting the structure of the maximally supersymmetric eleven dimensional plane-wave background, eq. (2.1). Its action is

$$S_{pp} = S_{\text{flat}} + S_\mu \,, \tag{2.2}$$

where each part of the action on the right hand side is given by

$$S_{\text{flat}} = \int dt \text{Tr}\left(\frac{1}{2R}D_tX^ID_tX^I + \frac{R}{4}([X^I,X^J])^2 + i\Theta^\dagger D_t\Theta - R\Theta^\dagger\gamma^I[\Theta,X^I]\right),$$
$$S_\mu = \int dt \text{Tr}\left(-\frac{1}{2R}\left(\frac{\mu}{3}\right)^2(X^i)^2 - \frac{1}{2R}\left(\frac{\mu}{6}\right)^2(X^a)^2 - i\frac{\mu}{3}\epsilon^{ijk}X^iX^jX^k - i\frac{\mu}{4}\Theta^\dagger\gamma^{123}\Theta\right). \quad (2.3)$$

Here, R is the radius of circle compactification along x^-, D_t is the covariant derivative with the gauge field A,

$$D_t = \partial_t - i[A,\ ]\,, \quad (2.4)$$

and γ^I is the 16×16 SO(9) gamma matrices. For practical study of the model, it is often convenient to make R disappear from the action by taking the rescaling of the gauge field and parameters as

$$A \to RA\,, \quad t \to \frac{1}{R}t\,, \quad \mu \to R\mu\,. \quad (2.5)$$

Then the actions in eq. (2.3) become

$$S_{\text{flat}} = \int dt \text{Tr}\left(\frac{1}{2}D_tX^ID_tX^I + \frac{1}{4}([X^I,X^J])^2 + i\Theta^\dagger D_t\Theta - \Theta^\dagger\gamma^I[\Theta,X^I]\right),$$
$$S_\mu = \int dt \text{Tr}\left(-\frac{1}{2}\left(\frac{\mu}{3}\right)^2(X^i)^2 - \frac{1}{2}\left(\frac{\mu}{6}\right)^2(X^a)^2 - i\frac{\mu}{3}\epsilon^{ijk}X^iX^jX^k - i\frac{\mu}{4}\Theta^\dagger\gamma^{123}\Theta\right), \quad (2.6)$$

which are free of R.

In matrix model, various objects, like branes and graviton, are realized by the classical solutions of the equations of motion for the matrix field. The dynamics between them is studied by expanding the matrix model action around the corresponding classical solution and performing the path integration. Let us denote the classical solution or the background configuration by B^I, and split the matrix quantities into as follows:

$$X^I = B^I + Y^I\,, \quad A = 0 + A\,, \quad \Theta = 0 + \Psi\,. \quad (2.7)$$

Then Y^I, A and Ψ are the quantum fluctuations around the background configuration, which are the fields subject to the path integration. We note that the gauge field may also have non-trivial classical configuration. However, it is simply set to zero in this paper because the objects we are interested in do not generate any background gauge field.

In taking into account the quantum fluctuations, we should recall that the matrix model itself is a gauge theory. This implies that the gauge fixing condition should be specified before proceed further. In this paper, we take the background field gauge which is usually chosen in the matrix model calculation,

$$D^{\text{bg}}_\mu A^\mu_{\text{qu}} \equiv D_tA + i[B^I,X^I] = 0\,. \quad (2.8)$$

Then the corresponding gauge-fixing S_{GF} and Faddeev-Popov ghost S_{FP} terms are given by

$$S_{\text{GF}} + S_{\text{FP}} = \int dt\, \text{Tr}\left(-\frac{1}{2}(D^{\text{bg}}_\mu A^\mu_{\text{qu}})^2 - \bar{C}\partial_tD_tC + [B^I,\bar{C}][X^I,C]\right)\,. \quad (2.9)$$

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Now by inserting the decomposition of the matrix fields (2.7) into eqs. (2.6) and (2.9), we get the gauge fixed plane-wave action S ($\equiv S_{pp} + S_{\rm GF} + S_{\rm FP}$) expanded around the classical background B^I. The resulting action is read as

$$S = S_0 + S_2 + S_3 + S_4 \,, \tag{2.10}$$

where S_n represents the action of order n with respect to the quantum fluctuations and, for each n, its expression is

$$\begin{aligned}
S_0 &= \int dt\, {\rm Tr}\left[\frac{1}{2}(\dot B^I)^2 - \frac{1}{2}\left(\frac{\mu}{3}\right)^2(B^i)^2 - \frac{1}{2}\left(\frac{\mu}{6}\right)^2(B^a)^2 + \frac{1}{4}([B^I,\, B^J])^2 - i\frac{\mu}{3}\epsilon^{ijk}B^iB^jB^k\right],\\
S_2 &= \int dt\, {\rm Tr}\left[\frac{1}{2}(\dot Y^I)^2 - 2i\dot B^I[A,\, Y^I] + \frac{1}{2}([B^I,\, Y^J])^2 + [B^I,\, B^J][Y^I,\, Y^J] - i\mu\epsilon^{ijk}B^iY^jY^k\right.\\
&\qquad - \frac{1}{2}\left(\frac{\mu}{3}\right)^2(Y^i)^2 - \frac{1}{2}\left(\frac{\mu}{6}\right)^2(Y^a)^2 + i\Psi^\dagger\dot\Psi - \Psi^\dagger\gamma^I[\Psi,\, B^I] - i\frac{\mu}{4}\Psi^\dagger\gamma^{123}\Psi\\
&\qquad \left. - \frac{1}{2}\dot A^2 - \frac{1}{2}([B^I,\, A])^2 + \dot{\bar C}\dot C + [B^I,\, \bar C][B^I,\, C]\right],\\
S_3 &= \int dt\, {\rm Tr}\Big[- i\dot Y^I[A,\, Y^I] - [A,\, B^I][A,\, Y^I] + [B^I,\, Y^J][Y^I,\, Y^J] + \Psi^\dagger[A,\, \Psi]\\
&\qquad - \Psi^\dagger\gamma^I[\Psi,\, Y^I] - i\frac{\mu}{3}\epsilon^{ijk}Y^iY^jY^k - i\dot{\bar C}[A,\, C] + [B^I,\, \bar C][Y^I,\, C]\Big],\\
S_4 &= \int dt\, {\rm Tr}\left[- \frac{1}{2}([A,\, Y^I])^2 + \frac{1}{4}([Y^I,\, Y^J])^2\right].
\end{aligned} \tag{2.11}$$

3 Background configuration

In this section, we set up the background configuration corresponding to the flat membrane and fuzzy sphere membrane, and discuss about the perturbation theory around it.

Since we will study the interaction between two objects, the matrices representing the background have the 2×2 block diagonal form as

$$B^I = \begin{pmatrix} B^I_{(1)} & 0 \\ 0 & B^I_{(2)} \end{pmatrix}, \tag{3.1}$$

where $B^I_{(s)}$ with $s = 1, 2$ are $N_s \times N_s$ matrices. If B^I are taken to be $N\times N$ matrices, then $N = N_1 + N_2$.

Basically, the configuration we consider is that the fuzzy sphere membrane is placed in the transverse space of the flat membrane with distance r. Each membrane is supposed to be supersymmetric. We would like to note that, unlike the case of membrane placed in flat space-time, supersymmetric membrane in plane-wave background may have a particular motion in a given situation. This feature stems from the nature of the plane-wave background.

The first object corresponding to $B^I_{(1)}$ is taken to be the fuzzy sphere membrane, which spans in SO(3) symmetric space and rotates in x^8-x^9 plane as follows:

$$\begin{aligned}
B^i_{(1)} &= \frac{\mu}{3}J^i\,,\\
B^8_{(1)} &= r\cos(\mu t/6)\mathbf{1}_{N_1\times N_1}\,, \qquad\qquad B^9_{(1)} = r\sin(\mu t/6)\mathbf{1}_{N_1\times N_1}
\end{aligned} \tag{3.2}$$

where J^i is in the N_1-dimensional irreducible representation of SU(2) and thus satisfies the SU(2) algebra,

$$[J^i, J^j] = i\epsilon^{ijk} J^k \ . \tag{3.3}$$

If the fuzzy sphere membrane sits at the origin in the SO(6) symmetric space, it preserves the full 16 dynamical supersymmetries of the plane-wave and hence is 1/2-BPS object. The above configuration contains a circular motion, and thus seems to break the supersymmetry. However, as has been shown explicitly in the path integral formulation [11], it is still supersymmetric basically due to the presence of the plane-wave background as alluded to above. In addition to this, the value of the classical action simply vanishes without any velocity dependent term. In this sense, the fuzzy sphere in circular motion may be regarded as a 'static' object.

The second object represented by $B^I_{(2)}$ is the flat membrane, which is taken to be the one found in [6]. It is 1/8-BPS object, and spans and spins in four dimensional subspace of the SO(6) symmetric space as

$$\begin{aligned} B^4_{(2)} &= Q\cos(\mu t/6)\,, & B^6_{(2)} &= Q\sin(\mu t/6)\,, \\ B^5_{(2)} &= P\cos(\mu t/6)\,, & B^7_{(2)} &= P\sin(\mu t/6)\,, \end{aligned} \tag{3.4}$$

where $N_2 \times N_2$ matrices, Q and P, satisfy

$$[Q, P] = i\sigma\,, \tag{3.5}$$

with a small constant parameter σ. We note that, in order to describe the flat membrane properly, the size of the matrix should be infinite. In what follows, N_2 is thus implicitly taken to be infinite. Now, from this somewhat complicated configuration, we see that, at $t = 0$, the flat membrane is placed in x^4-x^5 plane, and, as time goes by, one axis along x^4 rotates in x^4-x^6 plane while another axis along x^5 rotates in x^5-x^7 plane.

In spite of the circular motion of the fuzzy sphere membrane and the spinning motion of the flat membrane, the configuration of eq. (3.1) with eqs. (3.2) and (3.4) is similar to the configuration of D2 and D0 branes separated by a constant distance, because the distance between flat and fuzzy sphere membranes, r, does not change in time.

Having the background configuration (3.1), we first evaluate the classical value of the action S_0. Because all the motions involved in the background have the same period, $T = 12\pi/\mu$, it is sufficient to consider the action per one period, which is obtained as

$$S_0/T = -\frac{1}{2}N_2\sigma^2\,. \tag{3.6}$$

The next thing we are going to do in what follows is the computation of the one-loop correction to this action, that is, to the background, (3.2) and (3.4), due to the quantum fluctuations via the path integration of the quadratic action S_2, and obtain the one-loop effective action Γ_{eff} or the effective potential V_{eff}. Before doing the one-loop computation, it should be made clear that S_3 and S_4 of eq. (2.11) can be regarded as perturbations. For this purpose, following [3], we rescale the fluctuations and parameters as

$$\begin{aligned} &A \to \mu^{-1/2}A\,, \quad Y^I \to \mu^{-1/2}Y^I\,, \quad C \to \mu^{-1/2}C\,, \quad \bar{C} \to \mu^{-1/2}\bar{C}\,, \\ &r \to \mu r\,, \quad t \to \mu^{-1}t\,, \quad Q \to \mu Q\,, \quad P \to \mu P\,, \quad \sigma \to \mu^2\sigma\,. \end{aligned} \tag{3.7}$$

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Under this rescaling, the powers of μ are factored out from the action S in the background (3.2) and (3.4) as

$$S = \mu^3 S_0 + S_2 + \mu^{-3/2} S_3 + \mu^{-3} S_4 \,, \tag{3.8}$$

where S_0, S_2, S_3 and S_4 do not have μ dependence and the period of motion becomes 12π. Now it is obvious that, in the large μ limit, S_3 and S_4 can be treated as perturbations and the one-loop computation gives the sensible result.

Based on the structure of (3.1), we now write the quantum fluctuations in the 2×2 block matrix form as follows:

$$\begin{aligned}
A &= \begin{pmatrix} 0 & \Phi^0 \\ \Phi^{0\dagger} & 0 \end{pmatrix}, \qquad & Y^I &= \begin{pmatrix} 0 & \Phi^I \\ \Phi^{I\dagger} & 0 \end{pmatrix}, \qquad & \Psi &= \begin{pmatrix} 0 & \chi \\ \chi^\dagger & 0 \end{pmatrix}, \\
C &= \begin{pmatrix} 0 & C \\ C^\dagger & 0 \end{pmatrix}, \qquad & \bar{C} &= \begin{pmatrix} 0 & \bar{C} \\ \bar{C}^\dagger & 0 \end{pmatrix}. & &
\end{aligned} \tag{3.9}$$

Although we denote the block off-diagonal matrices for the ghosts by the same symbols with those of the original ghost matrices, there will be no confusion since $N \times N$ matrices will never appear from now on. The reason why the block-diagonal parts are not considered is that they do not give any effect on the interaction between two kinds of membranes but lead to the quantum correction to each membrane itself, which vanishes because the membranes considered here are supersymmetric.

4 One-loop path integration

We now perform the path integration for the action of quadratic fluctuations, S_2, around the classical background (3.1) with (3.2) and (3.4). The results will be formal and the actual evaluation of them for the effective action or potential will be described in the next section.

The quadratic action is largely composed of three decoupled sectors, which are bosonic, ghost, and fermionic sectors. In the path integration of each sector, the integration variables are matrices. For the actual evaluation of the path integration, it is usually useful to expand the matrix variables in a suitable matrix basis. Taking a matrix basis depends on the classical background under consideration. For example, in the study of fuzzy spheres, the matrix spherical harmonics provides a good matrix basis for the fluctuations around the configuration of fuzzy spheres. For the present case where the flat membrane is involved, the matrix spherical harmonics is not adequate, and we should look for another basis.

For taking a suitable matrix basis, we first consider the fluctuations around the fuzzy sphere and flat membranes separately. The fuzzy sphere of eq. (3.2) is described by N_1-dimensional or spin-j representation of SU(2) with

$$j = \frac{1}{2}(N_1 - 1) \,. \tag{4.1}$$

Thus, the fluctuations around the fuzzy sphere are naturally expressed in terms of the states $|m\rangle$ in the spin-j representation of SU(2), where $-j \le m \le j$. J^i describing fuzzy sphere acts on $|m\rangle$ in a standard way as

$$J^3|m\rangle = m|m\rangle \,, \quad J^\pm|m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|m\rangle \,, \tag{4.2}$$

where $J^{\pm} = J^1 \pm iJ^2$. As for the flat membrane of eq. (3.4), it has the characteristic given by the commutation relation, (3.5). If we define

$$a = \frac{1}{\sqrt{2\sigma}}(Q + iP)\,, \quad a^\dagger = \frac{1}{\sqrt{2\sigma}}(Q - iP)\,, \tag{4.3}$$

then they satisfy the commutation relation

$$[a, a^\dagger] = 1\,, \tag{4.4}$$

and can be regarded as the annihilation and creation operators of simple harmonic oscillator. This fact allows us to express the fluctuations around the flat membrane in terms of the oscillator states, on which a and $a^\dagger$ act as

$$a|n\rangle = \sqrt{n}|n-1\rangle\,, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle\,. \tag{4.5}$$

Because the size of the membrane is given by N_2, the oscillator number n runs from 0 to $N_2 - 1$, and hence has the upper bound. However, we note that actually there is no upper bound for n because N_2 should be infinite for the proper description of the flat membrane.

From the above consideration and the structure of eq. (3.9), the matrix basis for the fluctuation can be taken to be $|m\rangle\langle n|$, where $|m\rangle$ is the state in spin-j representation of SU(2) and $\langle n|$ is an oscillator state. Then, in this matrix basis, each fluctuation matrix has the following mode expansion

$$\Phi = \sum_{m=-j}^{j}\sum_{n=0}^{\infty}\phi_{mn}|m\rangle\langle n|\,. \tag{4.6}$$

This expansion now allows us to reduces the path integration of the matrix variable to that of the mode ϕ_{mn}.

4.1 Bosonic sector

The Lagrangian for the bosonic sector of the quadratic action S_2 is split into two parts

$$L_B = L_{\mathrm{SO}(3)} + L_{\mathrm{SO}(6)}\,, \tag{4.7}$$

where $L_{\mathrm{SO}(3)}$ is the Lagrangian for Φ^i and $L_{\mathrm{SO}(6)}$ is for Φ^0 and Φ^a. Because two parts are decoupled systems, each of them can be considered independently.

We fist deal with the path integration of $L_{\mathrm{SO}(3)}$. The Lagrangian is

$$L_{\mathrm{SO}(3)} = \mathrm{Tr}\left[|\dot{\Phi}^i|^2 - (r^2 + Q^2 + P^2)|\Phi^i|^2 - \frac{1}{3^2}|\Phi^i + i\epsilon^{ijk}J^j\Phi^k|^2 - \frac{1}{3^2}\Phi^{i\dagger}J^iJ^j\Phi^j\right]. \tag{4.8}$$

Due to the third term in the trace, the diagonalization of Φ^i is required. The procedure of diagonalization has been well established based on the mode expansion (4.6) and the standard SU(2) algebra [3, 11]. If we adopt the procedure with the same symbols used in previous literatures, the diagonalization of the modes of Φ^i, that is, ϕ^i_{mn}, leads to α_{mn}, β_{mn}, and ω_{mn}, which are described by the following Lagrangian.

$$
\begin{aligned}
L_{\mathrm{SO}(3)} = &\sum_{m=-j+1}^{j-1}\sum_{n=0}^{\infty}\left[|\dot\alpha_{mn}|^2 - \left(r^2+\sigma(2n+1)+\frac{1}{3^2}j^2\right)|\alpha_{mn}|^2\right] \\
&+ \sum_{m=-j-1}^{j+1}\sum_{n=0}^{\infty}\left[|\dot\beta_{mn}|^2 - \left(r^2+\sigma(2n+1)+\frac{1}{3^2}(j+1)^2\right)|\beta_{mn}|^2\right] \\
&+ \sum_{m=-j}^{j}\sum_{n=0}^{\infty}\left[|\dot\omega_{mn}|^2 - \left(r^2+\sigma(2n+1)+\frac{1}{3^2}j(j+1)\right)|\omega_{mn}|^2\right], \qquad (4.9)
\end{aligned}
$$

where the range of m has been changed due to the effect of diagonalization and

$$
(Q^2+P^2)|n\rangle = \sigma(2a^\dagger a+1)|n\rangle = \sigma(2n+1)|n\rangle \qquad (4.10)
$$

has been used. As noted in [3, 11], the mode ω_{mn} corresponds to the gauge degree of freedom and its effect should be cancelled by the contribution from ghosts. Now, having the fully diagonalized Lagrangian, it is straightforward to perform the path integration and get

$$
\prod_{n=0}^{\infty}\det{}^{-(2j-1)}\Delta_{(n)10}\cdot\det{}^{-(2j+3)}\Delta_{(n)11}\cdot\det{}^{-(2j+1)}\Delta_{(n)}\,. \qquad (4.11)
$$

where we have defined

$$
\begin{aligned}
\Delta_{(n)\alpha\beta} &\equiv -\partial_t^2 - r^2 - \sigma(2n+\alpha) - \frac{1}{3^2}(j+\beta)^2\,, \\
\Delta_{(n)} &\equiv -\partial_t^2 - r^2 - \sigma(2n+1) - \frac{1}{3^2}j(j+1)\,. \qquad (4.12)
\end{aligned}
$$

We turn to another part of the bosonic sector, which is given by

$$
\begin{aligned}
L_{\mathrm{SO}(6)} = \mathrm{Tr}\Big\{ &-|\dot\Phi^0|^2 + (r^2+Q^2+P^2)|\Phi^0|^2 + \frac{1}{3^2}\Phi^{0\dagger}J^iJ^i\Phi^0 \\
&+ |\dot\Phi^a|^2 - \left(r^2+Q^2+P^2+\frac{1}{6^2}\right)|\Phi^a|^2 - \frac{1}{3^2}\Phi^{a\dagger}J^iJ^i\Phi^a \\
&+ \frac{i}{3}\sin(t/6)\left[\Phi^{0\dagger}(\Phi^4 Q+\Phi^5 P - r\Phi^8) - (Q\Phi^{4\dagger}+P\Phi^{5\dagger}-r\Phi^{8\dagger})\Phi^0\right] \\
&- \frac{i}{3}\cos(t/6)\left[\Phi^{0\dagger}(\Phi^6 Q+\Phi^7 P - r\Phi^9) - (Q\Phi^{6\dagger}+P\Phi^{7\dagger}-r\Phi^{9\dagger})\Phi^0\right] \\
&+ 2i\sigma\Big[\left(\cos(t/6)\Phi^{4\dagger}+\sin(t/6)\Phi^{6\dagger}\right)\left(\cos(t/6)\Phi^5+\sin(t/6)\Phi^7\right) \\
&- \left(\cos(t/6)\Phi^{5\dagger}+\sin(t/6)\Phi^{7\dagger}\right)\left(\cos(t/6)\Phi^4+\sin(t/6)\Phi^6\right)\Big]\Big\}\,. \qquad (4.13)
\end{aligned}
$$

There are lots of trigonometric functions in this Lagrangian due to the motion of background membranes. The fact that they have explicit time dependence makes the path integration cumbersome. Thus, it is desirable to hide the explicit time dependence by

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taking some redefinition of matrix variables. For the present case, we take

$$\begin{aligned}
&\cos(t/6)\Phi^4+\sin(t/6)\Phi^6\to\Phi^4\,, && -\sin(t/6)\Phi^4+\cos(t/6)\Phi^6\to\Phi^6\,,\\
&\cos(t/6)\Phi^5+\sin(t/6)\Phi^7\to\Phi^5\,, && -\sin(t/6)\Phi^5+\cos(t/6)\Phi^7\to\Phi^7\,,\\
&\cos(t/6)\Phi^8+\sin(t/6)\Phi^9\to\Phi^8\,, && -\sin(t/6)\Phi^8+\cos(t/6)\Phi^9\to\Phi^9\,,
\end{aligned}\tag{4.14}$$

which is nothing but the transformation to the rotating frame. Then, under this transformation, the Lagrangian becomes

$$\begin{aligned}
L_{\mathrm{SO}(6)}=\mathrm{Tr}\Big\{&-|\dot\Phi^0|^2+(r^2+Q^2+P^2)|\Phi^0|^2+\frac{1}{3^2}\Phi^{0\dagger}J^iJ^i\Phi^0\\
&+|\dot\Phi^a|^2-(r^2+Q^2+P^2)|\Phi^a|^2-\frac{1}{3^2}\Phi^{a\dagger}J^iJ^i\Phi^a\\
&+\frac{1}{3}(\Phi^{4\dagger}\dot\Phi^6-\Phi^{6\dagger}\dot\Phi^4)+\frac{1}{3}(\Phi^{5\dagger}\dot\Phi^7-\Phi^{7\dagger}\dot\Phi^5)+\frac{1}{3}(\Phi^{8\dagger}\dot\Phi^9-\Phi^{9\dagger}\dot\Phi^8)\\
&-\frac{i}{3}\Phi^{0\dagger}(\Phi^6Q+\Phi^7P-r\Phi^9)+\frac{i}{3}(Q\Phi^{6\dagger}+P\Phi^{7\dagger}-r\Phi^{9\dagger})\Phi^0\\
&+2i\sigma(\Phi^{4\dagger}\Phi^5-\Phi^{5\dagger}\Phi^4)\Big\}\,,
\end{aligned}\tag{4.15}$$

which is obviously free of trigonometric functions having explicit time dependence.

Now, by using the mode expansion eq. (4.6) for each matrix variable, we can express this Lagrangian in terms of modes. We notice however that the terms linear in Q and P lead to coupling of modes with different oscillator number n because Q and P are linear combinations of the creation and annihilation operators as seen in eq. (4.3). In order to avoid such coupling, we follow the prescription given in [32] and define new matrix variables as

$$\Phi^\pm\equiv\frac{1}{\sqrt2}(\Phi^4\pm i\Phi^5)\,,\quad \tilde\Phi^\pm\equiv\frac{1}{\sqrt2}(\Phi^6\pm i\Phi^7)\,,\tag{4.16}$$

which is nothing but a unitary transformation. Then the terms linear in Q and P become

$$\begin{aligned}
&\frac{i}{3}\mathrm{Tr}\left\{-\Phi^{0\dagger}(\Phi^6Q+\Phi^7P)+(Q\Phi^{6\dagger}+P\Phi^{7\dagger})\Phi^0\right\}\\
&\qquad=\frac{i}{3}\sqrt{\sigma}\,\mathrm{Tr}\left\{-\Phi^{0\dagger}(\tilde\Phi^+a^\dagger+\tilde\Phi^-a)+(a\tilde\Phi^{+\dagger}+a^\dagger\tilde\Phi^{-\dagger})\Phi^0\right\}\,,
\end{aligned}\tag{4.17}$$

where eq. (4.3) has been used. This structure naturally leads us to take the mode expansions for $\Phi^\pm$ and $\tilde\Phi^\pm$ as

$$\Phi^\pm=\sum_{m=-j}^{j}\sum_{n=\mp1}^{\infty}\phi^\pm_{mn}|m\rangle\langle n\pm1|\,,\quad \tilde\Phi^\pm=\sum_{m=-j}^{j}\sum_{n=\mp1}^{\infty}\tilde\phi^\pm_{mn}|m\rangle\langle n\pm1|\,,\tag{4.18}$$

while Φ^0, Φ^8, and Φ^9 are taken to follow the expansion of eq. (4.6). We note that $\Phi^\pm$ and $\tilde\Phi^\pm$ should have the same type of mode expansion, since they couple to each other with one time derivative.

Having proper mode expansions for matrix variables, there is no longer mode mixing between different n or m, and thus the Lagrangian is the sum of parts each of which is labeled by m and n. For a given m and n, after some manipulation with eqs. (4.2) and (4.5), the part of the Lagrangian, say $L_{(mn)}$, is obtained as

$$L_{(mn)} = V^\dagger_{(mn)} M_{(mn)} V_{(mn)}\,, \tag{4.19}$$

where $V_{(mn)} = (\phi^0_{mn},\, \phi^+_{mn},\, \tilde\phi^+_{mn},\, \phi^-_{mn},\, \tilde\phi^-_{mn},\, \phi^8_{mn},\, \phi^9_{mn})^T$ and

$$M_{(mn)} = \begin{pmatrix} -\Delta_{(n)} & 0 & -\frac{i}{3}\sqrt{\sigma}\sqrt{n+1} & 0 & -\frac{i}{3}\sqrt{\sigma}\sqrt{n} & 0 & \frac{i}{3}r \\ 0 & \Delta_{(n)} & \frac{1}{3}\partial_t & 0 & 0 & 0 & 0 \\ \frac{i}{3}\sqrt{\sigma}\sqrt{n+1} & -\frac{1}{3}\partial_t & \Delta_{(n)}-2\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_{(n)} & \frac{1}{3}\partial_t & 0 & 0 \\ \frac{i}{3}\sqrt{\sigma}\sqrt{n} & 0 & 0 & -\frac{1}{3}\partial_t & \Delta_{(n)}+2\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{(n)} & \frac{1}{3}\partial_t \\ -\frac{i}{3}r & 0 & 0 & 0 & 0 & -\frac{1}{3}\partial_t & \Delta_{(n)} \end{pmatrix}, \tag{4.20}$$

where $\Delta_{(n)}$ has been defined in eq. (4.12). Before summing up $L_{(mn)}$ for m and n, we should notice that the oscillator number n of ϕ^+_{mn} and $\tilde\phi^+_{mn}$ starts from -1 while that of ϕ^-_{mn} and $\tilde\phi^-_{mn}$ starts from $+1$, as we can see from eq. (4.18). It is easy to see that the modes ϕ^+_{mn} and $\tilde\phi^+_{mn}$ at $n=-1$ are decoupled from other modes and form a subsystem, because all other modes do not have such oscillator number. As for the modes ϕ^-_{mn} and $\tilde\phi^-_{mn}$, the absence of them at $n=0$ seems to require an independent treatment of $M_{(m0)}$. However, let us suppose that these modes were present at the beginning. Then, the structure of $M_{(m0)}$ shows that they are decoupled from other modes and form a subsystem. Furthermore, the subsystem is exactly the same with that composed of ϕ^+_{mn} and $\tilde\phi^+_{mn}$ at $n=-1$. This indicates that the modes ϕ^+_{m-1} and $\tilde\phi^+_{m-1}$ can be symbolically identified with ϕ^-_{m0} and $\tilde\phi^-_{m0}$. More precisely, $\phi^+_{m-1} \to \tilde\phi^-_{m0}$ and $\tilde\phi^+_{m-1} \to \phi^-_{m0}$, which can be inferred from $M_{(m0)}$. After all, all the modes can be taken to have the oscillator number starting from $n=0$, and thus the Lagrangian $L_{\mathrm{SO}(6)}$ is written in terms of modes as

$$L_{\mathrm{SO}(6)} = \sum_{m=-j}^{j}\sum_{n=0}^{\infty} V^\dagger_{(mn)} M_{(mn)} V_{(mn)}\,. \tag{4.21}$$

From the above mode expanded Lagrangian $L_{\mathrm{SO}(6)}$, the formal evaluation of the path integral results in

$$\prod_{m=-j}^{j}\prod_{n=0}^{\infty} \mathrm{Det}^{-1} M_{(mn)}\,, \tag{4.22}$$

where Det involves the matrix determinant as well as the usual functional one. In order to get the one-loop effective action or potential, we should first diagonalize the matrix $M_{(mn)}$. However, the diagonalization of $M_{(mn)}$ is not an easy task, basically due to the two constant terms $\pm 2\sigma$ appearing in the diagonal elements of the matrix. Fortunately, if we consider $M_{(mn)}$ without these two terms, it can be diagonalized without much difficulty. This fact naturally leads us to consider a perturbation expansion in terms of σ. Actually, it is not necessary to diagonalize the matrix $M_{(mn)}$ exactly. We are interested in the membrane interaction in the long distance limit, and hence the perturbation expansion is enough for our purpose. Furthermore, since the constant parameter σ is a small quantity corresponding to the quantum of the area of flat membrane, it is a good expansion parameter.

If we denote $M_{(mn)}$ without $\pm 2\sigma$ in the diagonal elements as $M^{(0)}_{(mn)}$, then the determinant of $M_{(mn)}$ is written as

$$\mathrm{Det}^{-1} M_{(mn)} = \mathrm{Det}^{-1} M^{(0)}_{(mn)} \cdot \det{}^{-1}\left[1 + 2\epsilon \frac{E_{(n)}}{P_{(n)}}\right] \tag{4.23}$$

where $\epsilon \equiv \sigma^2$ for emphasizing the parameter of perturbative expansion,

$$\mathrm{Det}^{-1} M^{(0)}_{(mn)} = \det{}^{-1}\left[-\Delta_{(n)} P_{(n)}\right] , \tag{4.24}$$

and various quantities inside the functional determinants are defined by

$$\begin{aligned}
P_{(n)} &\equiv \Delta_{(n)10}\Delta_{(n)11}(\Delta_{(n)1\frac{1}{2}} + a_{n+})^2(\Delta_{(n)1\frac{1}{2}} + a_{n-})^2 , \\
E_{(n)} &\equiv \frac{1}{3^2}\Delta_{(n)}(\Delta_{(n)1\frac{1}{2}} + a_{n+})(\Delta_{(n)1\frac{1}{2}} + a_{n-}) - 2\Delta^2_{(n)}(\Delta_{(n)1\frac{1}{2}} + b_{n+})(\Delta_{(n)1\frac{1}{2}} + b_{n-}) , \\
a_{n\pm} &\equiv -\frac{1}{6^2} \pm \frac{1}{3}\sqrt{r^2 + \sigma(2n+1) + \frac{1}{3^2}\left(j + \frac{1}{2}\right)^2} , \\
b_{n\pm} &\equiv -\frac{1}{6^2} \pm \frac{1}{3}\sqrt{\sigma(2n+1) + \frac{1}{3^2}\left(j + \frac{1}{2}\right)^2} .
\end{aligned} \tag{4.25}$$

This expression of the determinant is of calculable form and can be studied perturbatively. Then the result of path integration for $L_{\mathrm{SO}(6)}$ now becomes

$$\prod_{n=0}^{\infty} \det{}^{-(2j+1)}\Delta_{(n)} \cdot \det{}^{-(2j+1)} P_{(n)} \cdot \det{}^{-(2j+1)}\left[1 + 2\epsilon\frac{E_{(n)}}{P_{(n)}}\right] . \tag{4.26}$$

4.2 Ghost sector

The ghost sector of the quadratic action S_2 is described by the Lagrangian

$$\begin{aligned}
L_G =& \mathrm{Tr}\Big[\dot{\bar{C}}^\dagger \dot{C} - (r^2 + Q^2 + P^2)\bar{C}^\dagger C - \frac{1}{3^2}\bar{C}^\dagger J^i J^i C \\
&+ \dot{\bar{C}}\dot{C}^\dagger - \bar{C}(r^2 + Q^2 + P^2)C^\dagger - \frac{1}{3^2}J^i J^i \bar{C} C^\dagger\Big] .
\end{aligned} \tag{4.27}$$

The path integration is carried out by using the same procedure taken in the previous subsection. If we denote the modes of the ghost variables C and $\bar{C}$ as c_{mn} and $\bar{c}_{mn}$ respectively, the Lagrangian in terms of modes is obtained as

$$L_G = \sum_{m=-j}^{j}\sum_{n=0}^{\infty}\left[\dot{\bar{c}}^*_{mn}\dot{c}_{mn} + \dot{\bar{c}}_{mn}\dot{c}^*_{mn} - \left(r^2 + \sigma(2n+1) + \frac{1}{3^2}j(j+1)\right)(\bar{c}^*_{mn}c_{mn} + \bar{c}_{mn}c^*_{mn})\right] . \tag{4.28}$$

The path integral for this Lagrangian is immediate, and evaluated as

$$\prod_{n=0}^{\infty} \det{}^{2(2j+1)}\Delta_{(n)} . \tag{4.29}$$

As it should be, this ghost contribution eliminates the contributions from unphysical gauge degrees of freedom in the results of bosonic sector, eqs. (4.11) and (4.26).

4.3 Fermionic sector

Finally, let us consider the fermionic sector of the quadratic action. Its Lagrangian is

$$\begin{aligned} L_F =& \mathrm{Tr}\Big[i\chi^\dagger\dot\chi - \frac{i}{4}\chi^\dagger\gamma^{123}\chi + \frac{1}{3}\chi^\dagger\gamma^i J^i\chi + r\chi^\dagger(\gamma^8\cos(t/6)+\gamma^9\sin(t/6))\chi \\ & - \chi^\dagger(\gamma^4\cos(t/6)+\gamma^6\sin(t/6))\chi Q - \chi^\dagger(\gamma^5\cos(t/6)+\gamma^7\sin(t/6))\chi P\Big], \end{aligned} \tag{4.30}$$

where the matrix variable χ has been rescaled by a factor $1/\sqrt{2}$. Due the periodic motion of background membranes, the Lagrangian has many trigonometric functions. Like we have done in the calculation of bosonic sector, we perform a transformation to the rotating frame

$$\chi \longrightarrow \Lambda\chi \tag{4.31}$$

using

$$\Lambda = e^{-\frac{1}{12}t\gamma^{46}}e^{-\frac{1}{12}t\gamma^{57}}e^{-\frac{1}{12}t\gamma^{89}}\,. \tag{4.32}$$

Under this transformation, the fermionic Lagrangian becomes

$$\begin{aligned} L_F =& \mathrm{Tr}\Big[i\chi^\dagger\dot\chi - \frac{i}{4}\chi^\dagger\gamma^{123}\chi + \frac{1}{3}\chi^\dagger\gamma^i J^i\chi + r\chi^\dagger\gamma^8\chi - \chi^\dagger(\gamma^4\chi Q + \gamma^5\chi P) \\ & - \frac{i}{12}\chi^\dagger(\gamma^{46}+\gamma^{57}+\gamma^{89})\chi\Big]. \end{aligned} \tag{4.33}$$

In the above Lagrangian, the term $\frac{1}{3}\chi^\dagger\gamma^i J^i\chi$ stems from the presence of the background fuzzy sphere and should be diagonalized. As in the case of the bosonic sector, the diagonalization can be carried out in exactly the same way considered in previous literatures [3, 11], and thus we will not repeat it here and just quote the result with brief explanation. Let us first take the mode expansion of χ according to eq. (4.6) as

$$\chi = \sum_{m=-j}^{j}\sum_{n=0}^{\infty}\chi_{mn}|m\rangle\langle n| \tag{4.34}$$

The mode χ_{mn} is a complex spinor with sixteen components, and in the representation **16** of SO(9). Under SO(9) $\to$ SO(3) $\times$ SO(6) $\simeq$ SU(2) $\times$ SU(4) reflecting the symmetry structure of the plane wave, χ is decomposed as $\mathbf{16} \to (\mathbf{2},\mathbf{4}) + (\bar{\mathbf{2}},\bar{\mathbf{4}})$. The diagonalization acts on the **2** and $\bar{\mathbf{2}}$ of SU(2), and results in two eigen-modes or eigen-spinors with eight independent components, say π_{mn} and η_{mn}, whose corresponding eigenvalues are $-j-1$ and j, respectively. Here, the range of m for π_{mn} (η_{mn}) is $-j \le m \le j-1$ ($-j-1 \le m \le j$). After this diagonalization, the Lagrangian L_F becomes the sum of two independent systems, which we call π-system, L_π, and η-system, L_η, and is given by

$$L_F = L_\pi + L_\eta\,, \tag{4.35}$$

where[1]

$$L_\pi = \sum_{m=-j}^{j-1} \mathrm{Tr}\bigg[i\pi_m^\dagger \dot{\pi}_m + \frac{i}{3}\left(j+\frac{1}{4}\right)\pi_m^\dagger \gamma^{123}\pi_m + r\pi_m^\dagger \gamma^8 \pi_m - \pi_m^\dagger(\gamma^4 \pi_m Q + \gamma^5 \pi_m P)$$
$$- \frac{i}{12}\pi_m^\dagger(\gamma^{46}+\gamma^{57}+\gamma^{89})\pi_m \bigg],$$
$$L_\eta = \sum_{m=-j-1}^{j} \mathrm{Tr}\bigg[i\eta_m^\dagger \dot{\eta}_m - \frac{i}{3}\left(j+\frac{3}{4}\right)\eta_m^\dagger \gamma^{123}\eta_m + r\eta_m^\dagger \gamma^8 \eta_m - \eta_m^\dagger(\gamma^4 \eta_m Q + \gamma^5 \eta_m P)$$
$$- \frac{i}{12}\eta_m^\dagger(\gamma^{46}+\gamma^{57}+\gamma^{89})\eta_m \bigg], \tag{4.36}$$

with the mode expansions

$$\pi_m = \sum_{n=0}^{\infty} \pi_{mn}\langle n|\,, \quad \eta_m = \sum_{n=0}^{\infty} \eta_{mn}\langle n|\,. \tag{4.37}$$

For the Lagrangians L_π and L_η, some comments are now in order. Firstly, in the mode expansions of π_m and η_m, we do not see the ket state $|m\rangle$ in the spin-j representation of SU(2) anymore. This is because the background effect due to the fuzzy sphere has been taken into account through the diagonalization. Secondly, there appears the term $i\pi_m^\dagger \gamma^{123}\pi_m$ in the π-system. This is also the case in the η-system. In the process of calculation, this term appears originally as $\pi_m^{(+)\dagger}\pi_m^{(+)} - \pi_m^{(-)\dagger}\pi_m^{(-)}$, where $\pi_m^{(+)}$ ($\pi_m^{(-)}$) is the variable coming from the diagonalization of $\mathbf{2}$ ($\bar{\mathbf{2}}$) of $(\mathbf{2},\mathbf{4})$ $((\bar{\mathbf{2}},\bar{\mathbf{4}}))$ in the decomposition of χ. Regarding to the action of $i\gamma^{123}$, $\pi_m^{(\pm)}$ satisfies $i\gamma^{123}\pi_m^{(\pm)} = \pm\pi_m^{(\pm)}$. This simply means that $\pi_m = \pi_m^{(+)} + \pi_m^{(-)}$ and hence we get $\pi_m^{(+)\dagger}\pi_m^{(+)} - \pi_m^{(-)\dagger}\pi_m^{(-)} = i\pi_m^\dagger\gamma^{123}\pi_m$.

By looking at the Lagrangians L_π and L_η of eq. (4.36), one can easily see that they have almost the same structure. The η-system can be obtained from π-system by changing the range of m and replacing j inside the trace by $-j-1$. Therefore, it is not necessary to consider the evaluation of path integral for both of them. From now on, we will focus on one system, say the π-system. The result for the η-system will follow naturally after completing the path integral of the π-system.

Before considering the path integral of the π-system, we would like to note that it is convenient to change the bra vector $\langle n|$ for the ket vector $|n\rangle$ in the mode expansion eq. (4.37). Such a change brings about some structural change inside the Lagrangian. More precisely,

$$\mathrm{Tr}\,\pi_m^\dagger(\gamma^4\pi_m Q + \gamma^5 \pi_m P) \;\longrightarrow\; \pi_m^\dagger(\gamma^4 Q - \gamma^5 P)\pi_m \tag{4.38}$$

under

$$\pi_m = \sum_{n=0}^{\infty}\pi_{mn}\langle n| \;\longrightarrow\; \pi_m = \sum_{n=0}^{\infty}\pi_{mn}|n\rangle\,, \tag{4.39}$$

[1] In previous works [3, 11, 13], the eight component spinor notation has been used. In this paper, we keep the sixteen component notation. So, π_{mn} and η_{mn} are sixteen component spinors but have only eight independent components.

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which can be easily checked by using eqs. (4.3) and (4.5). This makes the Lagrangian have more tractable form as follows.

$$L_\pi = \sum_{m=-j}^{j-1} \pi_m^\dagger \left[i\partial_t + \frac{i}{3}\left(j+\frac{1}{4}\right)\gamma^{123} + r\gamma^8 - \gamma^4 Q + \gamma^5 P - \frac{i}{12}(\gamma^{46}+\gamma^{57}+\gamma^{89})\right]\pi_m. \tag{4.40}$$

In the above Lagrangian, there are various products of gamma matrices. For treating them properly, we begin with the fact that π_{mn} has the positive chirality of SO(9) because it is in **16** of SO(9), that is, $\gamma_{(9)}\pi_{mn} = \pi_{mn}$ where $\gamma_{(9)} = \gamma^1\gamma^2\cdots\gamma^9$. If we consider the operator measuring the chirality in the SO(6) symmetric space as $\gamma_{(6)} = \gamma^4\gamma^5\gamma^6\gamma^7\gamma^8\gamma^9$, we see that $\gamma_{(9)} = \gamma^{123}\gamma_{(6)}$. This shows that, for a given SO(9) chirality, the eigenvalue of γ^{123} is automatically determined by that of $\gamma_{(6)}$, or vice versa. In succession, because $\gamma_{(6)} = -\gamma^{46}\gamma^{57}\gamma^{89}$, the chiralities in 4-6, 5-7, and 8-9 planes determine the eigenvalue of γ^{123}. Now, let us split π_m in terms of the chiralities in 4-6, 5-7, and 8-9 planes as

$$\pi_m = \sum_{s_1,s_2,s_3=\pm} \pi_{ms_1s_2s_3} = \sum_{n=0}^{\infty}\sum_{s_1,s_2,s_3=\pm} \pi_{mns_1s_2s_3}|n\rangle\,, \tag{4.41}$$

where s_1, s_2, and s_3 represent the eigenvalues of γ^{46}, γ^{57}, and γ^{89}, respectively. Then, the action of γ^{46} on $\pi_{mns_1s_2s_3}$ is given by

$$\gamma^{46}\pi_{mns_1s_2s_3} = is_1\pi_{mns_1s_2s_3}\,, \tag{4.42}$$

and similarly for γ^{57} and γ^{89}. As for the eigenvalue of γ^{123}, s_1, s_2, and s_3 determine it as

$$\gamma^{123} = -is_1s_2s_3\,. \tag{4.43}$$

In addition to the proper handling of products of gamma matrices, the presence of Q and P in the Lagrangian of eq. (4.40) leads to the mixing of modes with different oscillator number n. As we have done in the bosonic case, such mixing problem is cured by taking an appropriate unitary transformation and then newly defined mode expansions for some variables. We first consider the following unitary transformation.

$$\begin{aligned}
\zeta^\pm_{1m} &\equiv \frac{1}{\sqrt{2}}(\gamma^4\pi_{m+++} \pm i\gamma^5\pi_{m--+})\,,\\
\zeta^\pm_{2m} &\equiv \frac{1}{\sqrt{2}}(\gamma^4\pi_{m+--} \pm i\gamma^5\pi_{m-+-})\,,\\
\zeta^\pm_{3m} &\equiv \frac{1}{\sqrt{2}}(\gamma^4\pi_{m+-+} \pm i\gamma^5\pi_{m-++})\,,\\
\zeta^\pm_{4m} &\equiv \frac{1}{\sqrt{2}}(\gamma^4\pi_{m++-} \pm i\gamma^5\pi_{m---})\,.
\end{aligned} \tag{4.44}$$

These particular pairings are chosen such that the creation and annihilation operators $a^\dagger$ and a defined in eq. (4.3) appear independently in different terms. After the transformation, we find that $\zeta^\pm_{1m}$ and $\zeta^\pm_{3m}$ couple to each other as $-i\sqrt{\sigma}\zeta^{+\dagger}_{1m}a^\dagger\gamma^5\zeta^-_{3m} + i\sqrt{\sigma}\zeta^{-\dagger}_{1m}a\gamma^5\zeta^+_{3m}$ and its conjugation. $\zeta^\pm_{2m}$ and $\zeta^\pm_{4m}$ have the similar coupling. Like the case of eq. (4.18), the structure of couplings leads us to take the mode expansions for $\zeta^\pm_{2m}$ and $\zeta^\pm_{3m}$ as

$$\zeta^\pm_{2m} = \sum_{n=\mp 1}^{\infty} \zeta^\pm_{2mn}|n\pm 1\rangle\,, \quad \zeta^\pm_{3m} = \sum_{n=\mp 1}^{\infty} \zeta^\pm_{3mn}|n\pm 1\rangle\,, \tag{4.45}$$

while $\zeta^\pm_{1m}$ and $\zeta^\pm_{4m}$ are taken to have the standard mode expansion. Now, based on these mode expansions, we see that the Lagrangian of eq. (4.40) does not have any coupling between modes with different oscillator number, and is written as

$$L_\pi = \sum_{m=-j}^{j-1}\sum_{n=0}^{\infty} Z^\dagger_{(mn)} F_{(mn)} Z_{(mn)}\,, \tag{4.46}$$

where $Z_{(mn)} = (\zeta^+_{1mn}, \zeta^-_{1mn}, \zeta^+_{2mn}, \zeta^-_{2mn}, \zeta^+_{3mn}, \zeta^-_{3mn}, \zeta^+_{4mn}, \zeta^-_{4mn})^T$ and

$$F_{(mn)} = \begin{pmatrix} K_1 & 0 & \Gamma_n & D \\ 0 & K_2 & D & \Gamma^\dagger_n \\ \Gamma^\dagger_n & D & K_3 & 0 \\ D & \Gamma_n & 0 & K_4 \end{pmatrix}. \tag{4.47}$$

The various quantities inside the matrix $F_{(mn)}$ are 2×2 matrices and defined by

$$K_1 = \begin{pmatrix} i\partial_t + \frac{1}{3}(j+\frac{1}{2}) & \frac{1}{6} \\ \frac{1}{6} & i\partial_t + \frac{1}{3}(j+\frac{1}{2}) \end{pmatrix}, \quad K_2 = \begin{pmatrix} i\partial_t + \frac{1}{3}j & 0 \\ 0 & i\partial_t + \frac{1}{3}j \end{pmatrix},$$
$$K_3 = \begin{pmatrix} i\partial_t - \frac{1}{3}j & 0 \\ 0 & i\partial_t - \frac{1}{3}j \end{pmatrix}, \quad K_4 = \begin{pmatrix} i\partial_t - \frac{1}{3}(j+\frac{1}{2}) & \frac{1}{6} \\ \frac{1}{6} & i\partial_t - \frac{1}{3}(j+\frac{1}{2}) \end{pmatrix}, \tag{4.48}$$

and

$$\Gamma_n = \begin{pmatrix} 0 & -i\sqrt{2\sigma n}\gamma^5 \\ i\sqrt{2\sigma(n+1)}\gamma^5 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} r\gamma^8 & 0 \\ 0 & r\gamma^8 \end{pmatrix}. \tag{4.49}$$

We would like to note that, in writing the Lagrangian L_π of eq. (4.46), we have used the reasoning similar to that leading to $L_{\mathrm{SO}(6)}$ of eq. (4.21), and identified symbolically ζ^+_{2m-1} and ζ^+_{3m-1} with ζ^-_{2m0} and ζ^-_{3m0}. So, the summation for n starts from 0.

The path integration of the π-system is now evaluated as

$$\prod_{m=-j}^{j-1}\prod_{n=0}^{\infty} \mathrm{Det}F_{(mn)}\,. \tag{4.50}$$

Because of the presence of gamma matrices inside $F_{(mn)}$, the computation of matrix determinant should be performed by using the following matrix identity repeatedly.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & 1 \end{pmatrix}\begin{pmatrix} 1 & A^{-1}B \\ 0 & D - CA^{-1}B \end{pmatrix}. \tag{4.51}$$

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After a bit of long computation, $\mathrm{Det}F_{(mn)}$ is obtained as

$$\begin{aligned}\mathrm{Det}F_{(mn)} &= \det\left[Q_{(n)} + \frac{\epsilon}{3^2}(\partial_t^2 + r^2 + \frac{1}{3^2}j^2)\right] \\ &= \det Q_{(n)} \cdot \det\left[1 + \frac{\epsilon}{3^2}\frac{\partial_t^2 + r^2 + \frac{1}{3^2}j^2}{Q_{(n)}}\right],\end{aligned} \tag{4.52}$$

where $\epsilon \equiv \sigma^2$ as in the bosonic case and, by using eq. (4.12), we have defined

$$\begin{aligned}Q_{(n)} &\equiv \Delta_{(n)00}\Delta_{(n)20}(\Delta_{(n)1\frac{1}{2}} + c_{n+})(\Delta_{(n)1\frac{1}{2}} + c_{n-}), \\ c_{n\pm} &\equiv -\frac{1}{6^2} \pm \frac{1}{3}\sqrt{r^2 + \sigma(2n+1) + \frac{1}{3^2}\left(j + \frac{1}{2}\right)^2 + 3^2\sigma^2}.\end{aligned} \tag{4.53}$$

By using this functional determinant for a given m and n, we can give the result of path integration for the π-system as

$$\prod_{n=0}^{\infty} \det{}^{2j} Q_{(n)} \cdot \det{}^{2j}\left[1 + \frac{\epsilon}{3^2}\frac{\partial_t^2 + r^2 + \frac{1}{3^2}j^2}{Q_{(n)}}\right]. \tag{4.54}$$

Finally, we consider the path integration of the η-system. As mentioned earlier, the η-system is the same with the π-system if we change the range of m and take the replacement $j \to -j-1$. This means that we can get the result of path integral for the η-system without any further calculation. Then, from the result of π-system, eq. (4.54), we see that the path integration of the η-system leads to

$$\prod_{n=0}^{\infty} \det{}^{2j+2} \tilde{Q}_{(n)} \cdot \det{}^{2j+2}\left[1 + \frac{\epsilon}{3^2}\frac{\partial_t^2 + r^2 + \frac{1}{3^2}(j+1)^2}{\tilde{Q}_{(n)}}\right], \tag{4.55}$$

where

$$\tilde{Q}_{(n)} \equiv \Delta_{(n)01}\Delta_{(n)21}(\Delta_{(n)1\frac{1}{2}} + c_{n+})(\Delta_{(n)1\frac{1}{2}} + c_{n-}). \tag{4.56}$$

5 Effective potential

We have evaluated the path integral for the bosonic, ghost, and fermionic sectors in the last section, and obtained the functional determinants given in eqs. (4.11), (4.26), (4.29), (4.54), and (4.55). The multiplication of them now gives $\exp(i\Gamma_{\text{eff}}^{\text{1-loop}})$, where $\Gamma_{\text{eff}}^{\text{1-loop}}$ is the one-loop effective action describing the interaction between the fuzzy sphere and flat membranes. In this section, we obtain the one-loop effective potential V_{eff} from the effective action via the relation $\Gamma_{\text{eff}}^{\text{1-loop}} = -\int dt V_{\text{eff}}$.

As we have seen in the last section, some functional determinants obtained after the formal path integral are not of fully factorized form. Although it is so, they can be studied perturbatively in terms of the small parameter ϵ which is defined by σ^2. By the way, the structure of functional determinants containing ϵ tells us that the ϵ expansion is nothing but the large distance expansion. This matches precisely with our purpose, because our prime interest is the leading order effective potential in the large distance limit. Here we would like to note that the large distance means large r compared to the size N_1 of the fuzzy sphere, that is, $r \gg N_1$.

At this point, apart from the numerical factor, one may actually guess the form of the leading order potential for the background configuration considered here. The guess is that the potential is attractive and behaves as $1/r^5$ at the leading order. However, as we will see, the calculation leads to an unexpected result that the leading order behavior is not $1/r^5$ but $1/r^3$.

Then we first consider the effective potential at the lowest order in ϵ. From the functional determinants, we can obtain the following potential without much difficulty.

$$\begin{aligned}\sum_{n=0}^{\infty}\Bigg[&4j\sqrt{m_{10}^2}+2(2j+2)\sqrt{m_{11}^2}+2(2j+1)\sqrt{m_{1\frac{1}{2}}^2+\frac{1}{6^2}+\frac{1}{3}\sqrt{m_{1\frac{1}{2}}^2}}\\&+2(2j+1)\sqrt{m_{1\frac{1}{2}}^2+\frac{1}{6^2}-\frac{1}{3}\sqrt{m_{1\frac{1}{2}}^2}}-2j\sqrt{m_{00}^2}-2j\sqrt{m_{20}^2}\\&-(2j+2)\sqrt{m_{01}^2}-(2j+2)\sqrt{m_{21}^2}-2(2j+1)\sqrt{m_{1\frac{1}{2}}^2+\frac{1}{6^2}+\frac{1}{3}\sqrt{m_{1\frac{1}{2}}^2+3^2\sigma^2}}\\&-2(2j+1)\sqrt{m_{1\frac{1}{2}}^2+\frac{1}{6^2}-\frac{1}{3}\sqrt{m_{1\frac{1}{2}}^2+3^2\sigma^2}}\Bigg],\end{aligned} \tag{5.1}$$

where

$$m_{\alpha\beta}^2 \equiv r^2+\sigma(2n+\alpha)+\frac{1}{3^2}(j+\beta)^2\,. \tag{5.2}$$

The potential is expressed as an infinite sum over n. This may cause to worry about convergence. However, if we investigate the potential at large n, we find that it behaves as $n^{-3/2}$ and thus the summation is well-defined. The sum over n can be performed by adopting the Euler-Maclaurin formula

$$\sum_{n=0}^{\infty} f(n)=\int_0^{\infty}dx f(x)+\frac{1}{2}f(0)-\frac{1}{12}f'(0)+\frac{1}{720}f'''(0)+\dots \tag{5.3}$$

which is valid when f and its derivatives vanish at infinity. After the summation, if we expand the resulting potential in terms of large r, we obtain

$$\frac{N_1\sigma}{r}-\frac{1}{432}(24j^2+24j+13)\frac{N_1\sigma}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right), \tag{5.4}$$

where $N_1=2j+1$ has been used.

We turn to the effective potential at the first order in ϵ. Let us first consider the contribution from the bosonic part, that is, from eq. (4.26). From the relation

$$\det{}^a(1+A)=\exp[\,a\mathrm{tr}\ln(1+A)\,]=\exp[\,a\mathrm{tr}A-a\mathrm{tr}A^2/2+\dots] \tag{5.5}$$

where tr is the functional trace, we see that the relevant contribution to the effective potential is $-2iN_1\sigma^2\sum_{n=0}^{\infty}\mathrm{tr}E_{(n)}/P_{(n)}$. The trace calculation of this is transformed to an integration in momentum space. After evaluating the integration, the Euler-Maclaurin formula (5.3) and the expansion in terms of large r then lead us to have the following bosonic contribution to the effective potential at ϵ^1-order.

$$-2iN_1\sigma^2 \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{E_{(n)}}{P_{(n)}} = -\frac{N_1\sigma}{r} + \frac{1}{216}(12j^2+12j+1)\frac{N_1\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right), \tag{5.6}$$

where ω is the conjugate momentum of time t, and ∂_t^2 inside $E_{(n)}/P_{(n)}$ is understood to be replaced by $-\omega^2$. This contribution shows explicitly that the $1/r$ term on the right hand side exactly cancels that of the lowest order potential (5.4). Thus, up to this point, the leading order interaction for large r is of $1/r^3$ type.

Another contribution at the first order in ϵ comes from fermionic part given by eqs. (4.54) and (4.55). If we follow the same steps taken in the previous paragraph, we get the contributions from the π-system (4.54) as

$$-\frac{1}{108}\frac{j\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right), \tag{5.7}$$

and from the η-system as

$$-\frac{1}{108}\frac{(j+1)\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \tag{5.8}$$

Thus, the total contribution from the fermionic part to the effective potential is

$$-\frac{1}{108}\frac{N_1\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \tag{5.9}$$

If we gather the results obtained up to now, eqs. (5.4), (5.6), and (5.9), then we see that the one-loop effective potential in the large distance limit becomes

$$V_{\text{eff}} = -\frac{5}{144}\frac{N_1\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \tag{5.10}$$

This is the effective potential up to the first order in ϵ. Here, one may wonder if the contributions coming from higher ϵ order correct the numerical factor of the leading order term or make r^{-5} the leading interaction term for large r by canceling the r^{-3} term in (5.10). However, if we contemplate eqs. (4.26), (4.54), and (4.55) and perform a simple power counting, it is not difficult to see that the higher ϵ order leads to at most the interaction of $\mathcal{O}(r^{-5})$. Therefore, the leading r^{-3} type interaction of (5.10) remains intact even if we consider the contributions from higher ϵ order, and it is one-loop exact.

The one-loop effective potential (5.10) shows that there is an interaction between the fuzzy sphere and flat membranes, which is attractive. At this point, let us recall the background configuration, (3.2) and (3.4). Although it is taken such that the fuzzy sphere membrane moves around the flat one, it is basically a 'static' one in a sense that the distance r between two membranes does not change as time goes by. The presence of an attractive interaction in this 'static' configuration strongly suggests that our membrane configuration is quite similar to the usual D2-D0 system where two D-branes are apart with a distance r. Since D0-brane is simply a graviton from the eleven-dimensional viewpoint, what we can conclude from this similarity is that the fuzzy sphere membrane behaves like a graviton,

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that is, a giant graviton. Thus the present calculation gives one more check about the interpretation of the fuzzy sphere membrane as a giant graviton.

One interesting fact is that the leading order interaction at large distance is of r^{-3} type rather than r^{-5} type. Usually, the increase of r power is related to the delocalization or smearing of brane in some directions. As for the present case, the r power increases by two from the expected power. This implies that one of two membranes is delocalized in two spatial directions. From the background configuration, it is not so difficult to guess that the flat membrane corresponds to such delocalized brane. The flat membrane of eq. (3.4) is taken to span and spin in four dimensions. So two extra directions are required for its description. We interpret that this brings about the delocalization or smearing effect which manifests in the interaction potential.

6 Conclusion and discussion

We have studied the interaction between flat and fuzzy sphere membranes in plane-wave matrix model and computed the one-loop effective potential at large distance limit. Similar to the usual D2-D0 system or more directly the membrane-graviton system in eleven dimensions [32], the interaction is non-vanishing and attractive. This shows that the fuzzy sphere membrane behaves like a graviton, the giant graviton. So, our result gives one more evidence about the interpretation of fuzzy sphere membrane as a giant graviton. By the way, interestingly enough, the leading interaction at large distance r is not the expected r^{-5} but r^{-3} type. We have interpreted this type of interaction as that incorporating the delocalization or smearing effect due to the configuration of the flat membrane which spans and spins in four dimensional space.

In fact, the smearing effect has been already reported in the supergravity side [35, 36]. In the plane-wave background, it has been observed that some supergravity solutions show the delocalization or smearing of branes in some directions. Our result may be the first explicit realization of the smearing effect in the matrix model side.

The effective potential we have obtained gives an attractive interaction. So, it is natural to expect that the final configuration may be the bound state of the flat and fuzzy sphere membranes. Although our effective potential is valid only at large distance and we do not know what happens at small distance, the bound state is quite interesting if it is possible. As for the D2-D0 system, two D-branes form a bound state at the final stage and D0-brane is realized as the magnetic field on the worldvolume of D2-brane. Contrary to the D0-brane, the fuzzy sphere membrane is not point-like and has a size. If it is really bound to the flat membrane, it is very interesting to ask about the fate of two membranes. At present, this is an open question. We hope to return to this issue in a near future.

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